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Threshold Transitions and Expertise Reversal: The Impact of Procedural Mastery on Critical Thinking in Pre-Service Mathematics Teachers

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Abstract

Background: Procedural mastery serves as a foundational component in the development of advanced critical thinking within educational ecosystems where teacher-training frameworks are continually developing. However, rigid instructional designs and automated cognitive patterns persist, heavily shaping pedagogical content knowledge and profoundly influencing the problem-solving capacity of future educators.

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Objective: This study examines how procedural-conceptual mastery limits critical thinking performance in geometric sequences among pre-service mathematics teachers, while evaluating the structural boundaries of the expertise reversal effect and non-linear threshold transitions.

Methodology: The empirical analysis utilises a sequential mixed-methods explanatory design combining qualitative and quantitative frameworks. A sample of 29 pre-service mathematics teachers was selected from public university education programmes in Indonesia. Quantitative data were operationalised via a paper-based assessment that differentiated between pre-critical performance (procedural fluency and basic conceptual recall) and critical thinking capabilities (modelling, interpretation, justification, and strategic decision-making), with qualitative insights captured through comprehensive written solutions and protocol-driven think-aloud interviews.

Results: The findings indicate that a strong performance asymmetry exists within the sample, demonstrating that pre-critical thinking scores are significantly more stable (M = 35.99/50; CV = 22.3%) than higher-order critical thinking scores (M = 22.50/50; CV = 71.0%). In opposition to assumptions of linear learning models, pre-critical and critical thinking do not report strong positive linear associations (r = -0.34), whereas overall test scores remain strongly correlated with critical thinking performance (r = 0.84). Concurrently, score distributions suggest a distinct cognitive missing middle, with no participants falling within intermediate performance bands. Qualitative theme extraction reveals that over-automated formulas function as a procedural knowledge trap that actively weakens structural reasoning and obscures justificatory control during advanced geometric sequence tasks.

Conclusion: The study concludes that procedural fluency assumes control over complex reasoning processes and constrains critical thinking in advanced learning contexts. Structural focus on algorithmic automaticity yields suboptimal cognitive outcomes when pre-service teachers encounter non-linear conceptual thresholds that require flexible execution.

Unique Contribution: This research advances cognitive load theory and mathematics teacher education literature by providing an integrated empirical framework that links the expertise reversal effect directly to macro-level threshold transitions within an emerging economy context, explicitly illustrating how rigid algorithmic architectures disrupt the development of deeper critical mathematical thinking.

Key Recommendation: It is recommended that curriculum designers, mathematics educators, and university instruction boards in teacher training institutions design educational reforms that discourage rote algorithmic memorisation. Additionally, policymakers should implement targeted instructional strategies that emphasize procedural flexibility, de-automatisation paradigms, and explicit justification practices to safeguard conceptual transparency and enhance the critical thinking capabilities of future mathematics teachers.

Keywords: procedural mastery; critical thinking; expertise reversal effect; threshold transitions; mathematics teacher education

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Introduction

Critical thinking is recognised as a key objective of contemporary mathematics education (Sachdeva & Eggen, 2021), particularly in the preparation of future teachers, who are expected to perform procedures accurately, interpret situations, justify decisions, and evaluate statements (Noviyanti et al., 2025). Despite this emphasis, instructional practices and curriculum design are shaped by a long-standing assumption that procedural mastery supports higher-order mathematical cognition (Clements & Sarama, 2025). This sequential model, progressing from procedural fluency to conceptual understanding and critical reflection, has shaped mathematics curricula worldwide and has been supported by meta-analytic research indicating a positive association between procedural competence and achievement (Djam'an et al., 2026; Gilmore et al., 2017; Rittle-Johnson & Schneider, 2015). However, results from cognitive psychology show that the correlation between procedural mastery and higher-order thinking is not necessarily facilitative. The expertise reversal effect has shown that instructional supports can become ineffective or counterproductive as students become more expert (Sudirman et al., 2025). In these circumstances, highly accomplished students may revert to procedural answers in tasks requiring modelling, justification, or evaluative judgments (Noviyanti et al., 2025). Students with high procedural fluency often show worse performance on non-routine tasks requiring adaptive expertise.

Similar issues have been observed in mathematics education, with students taught primarily through procedure-focused instruction reporting weaknesses in logical reasoning and explanation (Patac & Patac, 2025). Cross-cultural research shows that high procedural performance is not consistently associated with high mathematical reasoning (Schulz, 2024). Recent research further separates procedural efficiency from flexibility, reporting that strategy selection, comparison, and justification are more strongly associated with meaningful comprehension than speed or precision alone. Cognitively, over-automatisationcan limit students' willingness to question assumptions or consider alternative representations, constraining higher-order thinking (Chang et al., 2017; Van Hoof et al., 2017).

Exploring students' thought processes is essential for understanding how procedural fluency limits critical thinking. Harel's (2008) framework emphasised that mathematical learning involves the development of habitual reasoning orientations that shape task perception and problem-solving regulation. These patterns are also apparent in technology-based learning settings where tool dependency can affect structural reasoning and reflective judgment (Kandaga et al., 2025). Although learning is often conceptualised as progressive, recent perspectives on threshold transitions suggest that concept development can be discontinuous, involving periods of stalled learning followed by qualitative reorganisations of reasoning (Hurrell, 2021; Isnawan et al., 2026; Van Hoof et al., 2017).

These gaps are particularly evident in the study of geometric sequences, a topic that demands both procedural fluency and control over ratio structures, indexing, contextual meaning, and justification. For pre-service teachers, unresolved tensions between procedural mastery and critical thinking carry particular significance, as their reasoning patterns can directly shape future instructional practices. Therefore, this research aims to investigate the conditions and mechanisms through which procedural-conceptual mastery constrains critical thinking in geometric sequences among pre-service mathematics teachers. Guided by an expertise reversal perspective and informed by threshold transition theory, a sequential explanatory mixed-methods design is used to

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examine (a) the direction and patterns of the relationship between procedural and critical thinking performance, and (b) the reasoning processes through which students maintain procedural dominance or reorganise thinking across a conceptual threshold. The research question is as follows: How does the degree of procedural-conceptual mastery correlate with the success of pre-service teachers in developing critical thinking competence in geometric sequences?

Methodology

Design Research

An explanatory sequential mixed-methods design was used to examine when and how procedural-conceptual mastery limits critical thinking in geometric sequences among pre-service mathematics teachers. This design was preferred over purely quantitative approaches because identifying the conditions under which procedural mastery constrains reasoning requires not only statistical patterns but also in-depth examination of the cognitive mechanisms involved. In the quantitative phase, students completed a paper-based test yielding individual scores on two domains: Pre-Critical Thinking (procedural and basic conceptual performance) and Critical Thinking (modelling, interpretation, justification, and decision-making). Quantitative findings then informed the qualitative phase, in which cases showing a mismatch between high procedural proficiency and low critical thinking performance were selected for think-aloud protocols and semi-structured interviews. This integration enabled explanations at the level of underlying reasoning mechanisms rather than correlational patterns alone.

Context and Participants

The research took place within a mathematics teacher education programme during a planned instructional unit on geometric sequences, where data were collected under normal classroom conditions. The quantitative cohort comprised 29 fifth-semester pre-service mathematics teachers who had completed coursework in Basic Mathematics, Discrete Mathematics, and secondary mathematics curriculum content, confirming sufficient procedural and conceptual exposure. Consequently, difficulties with geometric sequence tasks were interpreted as constraints in balancing procedures with structural reasoning rather than prior knowledge deficits. Participation was voluntary with no impact on course grades. Although the single-institution sampling strengthens internal comparability, it raises the possibility that the procedural trap observed may partly reflect the local curriculum's emphasis on formula-based instruction rather than a purely cognitive phenomenon. Specifically, participants may have been pre-conditioned toward procedural dominance through prior coursework that systematically prioritised formula execution over sense-making and justification, a pre-conditioning that cannot be fully disentangled from individual cognitive tendencies in the present design. Future research should therefore replicate this study across institutions with contrasting pedagogical orientations to determine whether the procedural–critical gap reflects an institution-specific pattern or a more general mechanism in mathematics teacher education. For the qualitative phase, participants were purposively selected based on visible signs of a procedural knowledge trap, including rigid formula application, inappropriate indexation, or failure to connect numerical results to contextual meaning.

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Instrument and Data Collection

A paper-based test on geometric sequences assessed procedural-conceptual mastery and critical thinking across two domains. The Pre-Critical Thinking domain (5 items, 50 points) covered routine tasks completable through rule application with minimal contextual interpretation. The Critical Thinking domain (2 items, 50 points) included non-routine tasks requiring modelling, evaluation, and justification: (a) a business modelling problem involving declining sales and fixed operating costs, and (b) Zeno's Paradox (Achilles–tortoise problem). Item design integrated Southworth's Cognitive Transformation to Critical Thinking framework and CCTST subskills, as detailed in Table 1, with psychometric analysis confirming item validity (0.58) and internal consistency ($\omega = 0.84$).

Table 1: Indicator Blueprint for Pre-Critical and Critical Thinking Tasks

Domain	Task Summary	CCTST Subskills
Pre-Critical Thinking	Sum of the first six terms of a geometric sequence with first term 16	Interpretation; Analysis; Deductive reasoning
Pre-Critical Thinking	Given ($U_5 = 162$) and ($U_2 = -6$), determine the common ratio	Interpretation; Analysis; Inference
Pre-Critical Thinking	Determine parameter(s) in the general term (Un) from two conditions	Analysis; Deductive reasoning; Explanation
Pre-Critical Thinking	Three numbers form a geometric progression; the sum is 26, and the product is 216	Analysis; Inference; Deductive reasoning
Pre-Critical Thinking	Select a parameter where the resulting value is closest to 59,000	Interpretation; Analysis; Inference
Critical Thinking	Business modeling: declining sales with fixed operating costs; decide when to stop production	Interpretation; Analysis; Evaluation; Explanation; Inference; Deductive & Inductive reasoning
Critical Thinking	Zeno's Paradox (Achilles–tortoise): determine whether Achilles overtakes the tortoise	Interpretation; Analysis; Explanation; Inference; Deductive reasoning

The test was administered in a single 120-minute session under standard classroom conditions without notes or calculators. For the qualitative phase, individual think-aloud protocols (30–45 minutes) were conducted within one week, audio-recorded and transcribed verbatim, followed by semi-structured interviews on reasoning strategies, answer-completeness criteria, and evaluative task awareness. Cases exhibiting procedural knowledge traps — strong procedural performance paired with weak critical thinking — were selected for in-depth qualitative analysis.

Data Analysis

Data analysis followed three sequential steps: (1) quantitative discovery of performance patterns where procedural mastery constrained critical thinking, (2) qualitative description of underlying reasoning mechanisms, and (3) integrative interpretation supporting meta-inferences about

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expertise reversal and threshold transitions. Written responses were scored independently across Pre-Critical and Critical Thinking domains (0–50 each) as the primary quantitative data. Descriptive statistics and 95% confidence intervals were computed for each domain, with score relationships examined using Pearson's correlation supplemented by Spearman's rho and robust skipped correlations. Restricted cubic spline regression assessed linear and non-linear associations. Internal consistency was estimated using McDonald's ω, preferred over Cronbach's alpha, as it accommodates a congeneric measurement model, yielding more accurate reliability estimates without assuming tau-equivalence.

Qualitative analysis employed Reflexive Thematic Analysis on think-aloud transcripts and interview responses, combining inductive and deductive coding approaches. Two team members independently generated and clustered codes related to reasoning control, procedural stopping points, and justificatory engagement, with inter-rater reliability confirmed through Cohen's kappa ($\kappa = 0.78$). Analytic decisions were documented through reflexive memos and audit trails. The integration of quantitative and qualitative data enabled a coherent explanation of expertise reversal and threshold transition effects, as illustrated in Figure 1.

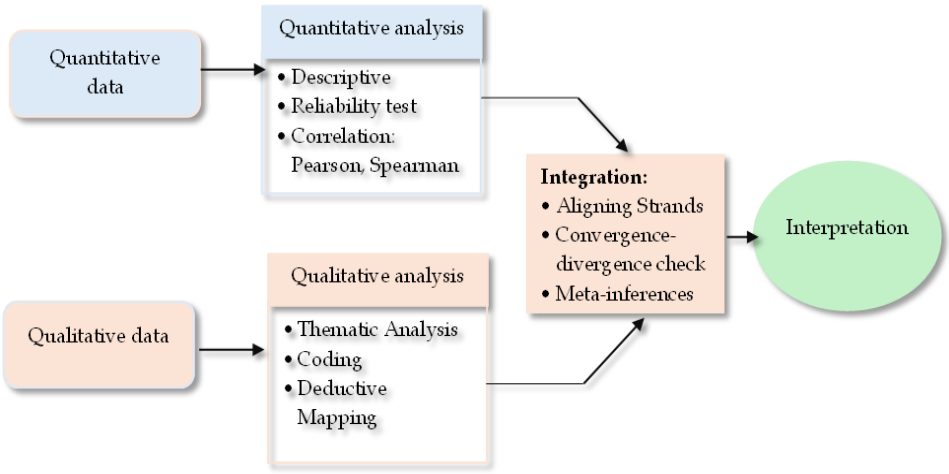

Figure 1. Research Procedures and Data Integration

Research Ethics

This research received ethical approval from Universitas Terbuka, Indonesia (486/U31.LPPM/DI.04.01/2025). Participants were informed of the study's purpose, risks, and confidentiality protections. Participation was voluntary, unrelated to course grades, and conducted under written informed consent with no monetary compensation.

Results and Discussion

Quantitative Performance Patterns Revealing a Procedural–Critical Thinking Gap

This section presents quantitative data from 29 pre-service mathematics teachers to identify performance conditions under which procedural-conceptual mastery does not support critical thinking in geometric sequences.

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Table 2: Descriptive Statistics of Cognitive Performance in Geometric Sequences Learning (N = 29)

Metric	Total Score	Pre-Critical Thinking	Critical Thinking
Mean	56.96	35.99	22.5
SD	16.43	8.03	15.97
Range	24.56–94.24	19.61–50.67	0.00–45.00
Percentage	57.00%	72.00%	45.00%

As shown in Table 2, students scored significantly higher in Pre-Critical Thinking tasks (M = 35.99, SD = 8.03; 72% achievement) than in Critical Thinking tasks (M = 22.50, SD = 15.97; 45% achievement), which provided a gap of 27-percentage-points. The overall level of performance (M = 56.96) indicates moderate success. However, a pronounced internal asymmetry becomes evident when performance is examined by cognitive domain. A general level of procedural and basic conceptual competence was achieved by the cohort. However, proficiency in critical thinking was not distributed uniformly and remained poorly developed.. This imbalance shows that critical thinking is not developed as a direct extension of procedural mastery.

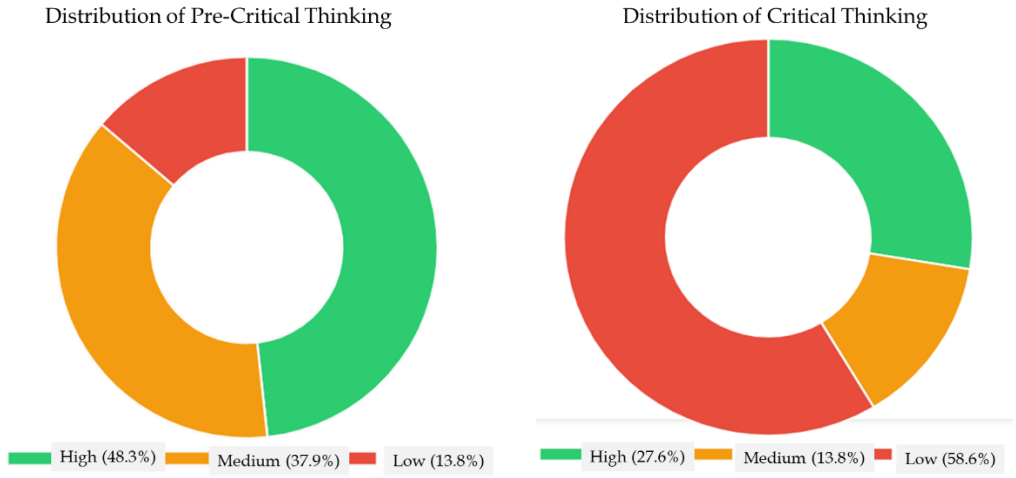

Figure 2. Performance Distribution Comparison

Variability patterns further enhance this interpretation. Pre-Critical Thinking scores showed low dispersion (CV = 22.3%), reflecting uniform procedural-conceptual proficiency across the cohort. By contrast, Critical Thinking scores were highly variable (CV = 71.0%), with performance spread from floor to near-ceiling, as illustrated in Figure 2. The similarity between the mean and standard deviation for Critical Thinking (22.50 ± 15.97) suggests a floor effect, indicating that a substantial portion of students were largely unable to engage meaningfully with critical thinking demands.

Table 3: Categorical Performance Analysis by Cognitive Domain

Variable	High (>75%)	Middle (50-75%)	Low (<50%)
Pre-Critical Thinking	14 (48.3%)	11 (37.9%)	4 (13.8%)
Critical Thinking	8 (27.6%)	4 (13.8%)	17 (58.6%)

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As reported in Table 3, 48.3% of the cohort achieved high Pre-Critical Thinking scores compared to only 27.6% in Critical Thinking, while 58.6% scored low in Critical Thinking against only 13.8% in Pre-Critical Thinking. This categorical imbalance indicates a qualitative gap between procedurally dominant students and those capable of restructuring thinking under critical demands. Notably, 4 students (13.8%) scored zero on Critical Thinking despite sufficient procedural-conceptual competence, confirming that difficulties cannot be attributed to prior knowledge deficits. This pattern reflects a disruption in cognitive coordination where procedural fluency ceases to serve as a resource for interpretation, evaluation, and justification — providing the first quantitative indication of a procedural–critical gap explored further through qualitative evidence.

Procedural Mastery and Critical Thinking: Correlational Patterns

This subsection examines when procedural–conceptual mastery ceases to support and begins to constrain critical thinking, using correlational analyses to identify patterns signalling a reversal in the functional role of procedural knowledge.

Table 4: Correlation Analysis between Cognitive Domains in Geometric Sequences Learning

Related Variables	Correlation (r)	Interpretation
Pre-Critical ↔ Critical	−0.34	Weak negative
Total ↔ Critical	0.839	Very strong positive
Total ↔ Pre-Critical	0.106	Very weak

As reported in Table 4, Pre-Critical and Critical Thinking scores exhibited a weak negative correlation ($r = -0.340$), indicating that procedural proficiency may actively suppress critical reasoning. Two mechanisms explain this: the expertise reversal effect (Kalyuga et al., 2012) posits that highly automatized knowledge dominates attentional resources rather than scaffolding interpretation; and over-automatisationsuppresses metacognitive monitoring (Schoenfeld, 2016; Seufert, 2018), reducing students' tendency to question assumptions. This inversion is confirmed by the broader relational structure — overall performance correlated strongly with Critical Thinking ($r = 0.839$) but negligibly with Pre-Critical Thinking ($r = 0.106$), suggesting functional decoupling between procedural mastery and higher-order reasoning.

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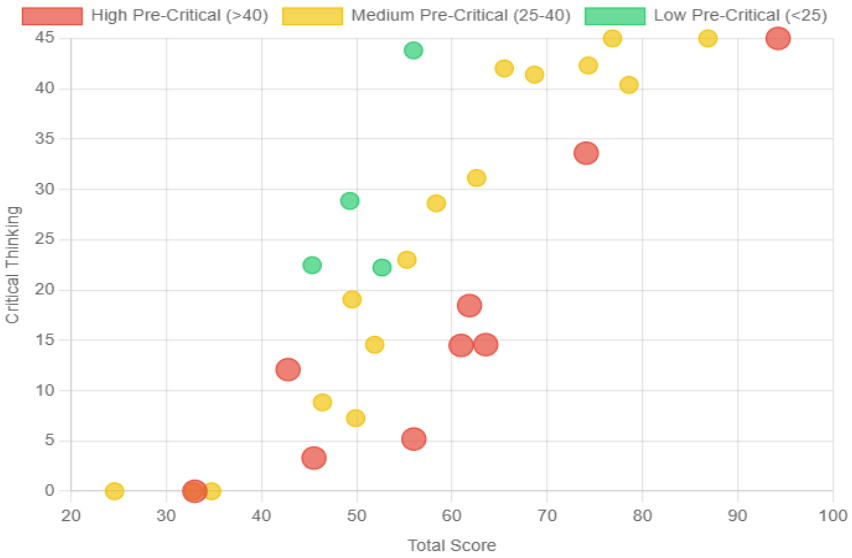

Figure 3. Relationship between Total Score and Critical Thinking Performance by Pre-Critical Thinking Level

Figure 3 plots Critical Thinking against Total Score by Pre-Critical Thinking levels, revealing an inverted-U trajectory via LOESS regression: Critical Thinking increases modestly from low to moderate procedural scores before declining at ceiling-level mastery. The negative slope for the high Pre-Critical Thinking group confirms that additional procedural proficiency is associated with lower critical thinking performance, with moderate procedural competence (25–40 points) outperforming ceiling-level mastery, constituting quantitative evidence of an expertise reversal effect.

Transition Patterns and the Missing Middle

The third research objective examines the extent to which the transition from procedural-conceptual mastery to critical thinking unfolds gradually or includes a qualitative shift. Figure 4 shows Critical Thinking scores with a clear bimodal distribution. The missing middle is the focus of a systematic gap, when and why procedural mastery limits critical thinking in geometric sequences.

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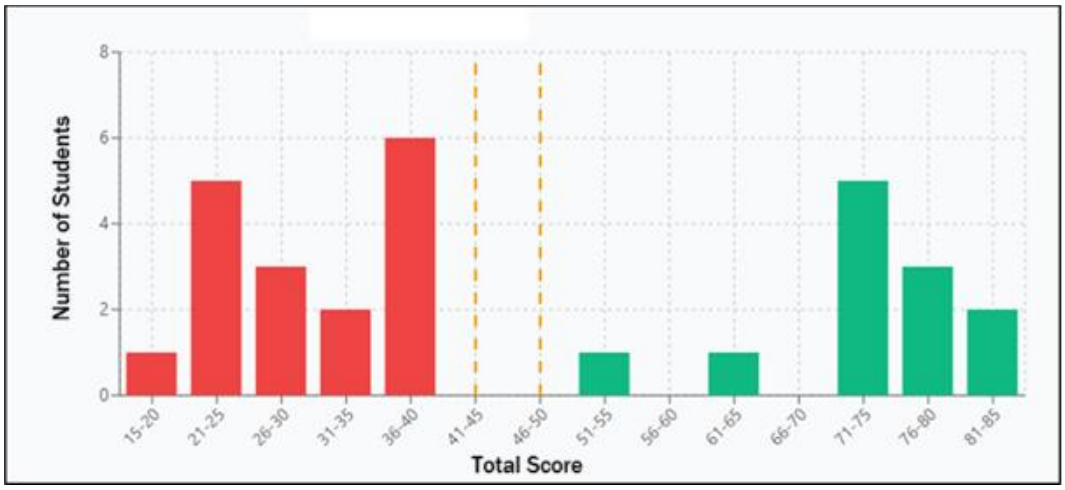

Figure 4. Histogram of Bimodal Distribution

The lower cluster (16 students, 55.2%; scores 0–40) represented a pre-transition state of procedural competence without critical reasoning, while the upper cluster (13 students, 44.8%; scores 52–81) indicated stable post-transition critical thinking. Notably, no students scored in the 41–51 range, confirming the absence of any intermediate developmental state.

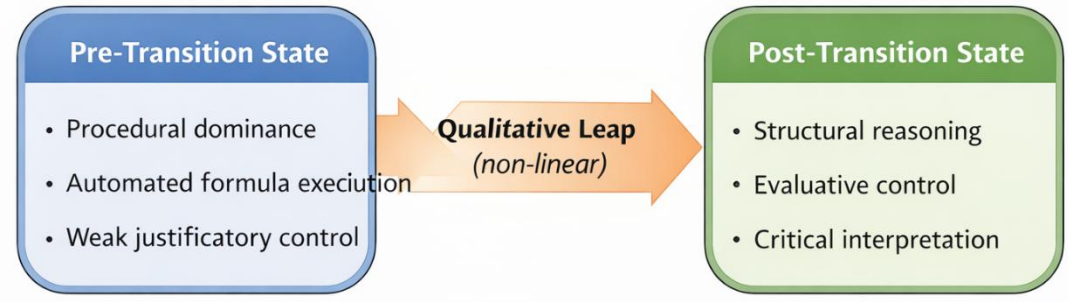

Figure 5. Threshold Transition Model from Procedural Dominance to Critical Thinking

Figure 5 illustrates that the shift from procedural competence to critical thinking operates as a qualitative jump rather than a gradual continuum. In the pre-transition state, students remain cognitively anchored to formula-based reasoning with limited structural interpretation, while the post-transition state is characterised by reorganisation of reasoning control, where procedures become subordinated to evaluative processes. No intermediate state was observed, consistent with threshold transition theory. Contrary to expectations, procedural growth produced no incremental gains — students either remained in the pre-transition state or moved directly to post-transition level, confirming that critical thinking requires qualitative restructuring rather than procedural accumulation alone.

Qualitative Findings

Procedural Dominance as a Stopping Rule for Reasoning

The first mechanism concerns how procedural mastery instigates premature reasoning termination, exemplified by K.D.S., who scored perfectly on procedural tasks (50/50) yet achieved only 12/50 in Critical Thinking.

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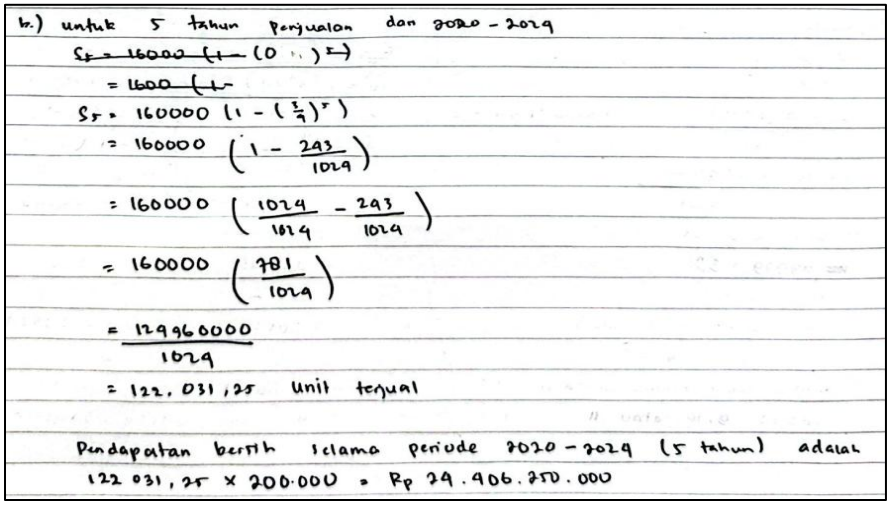

Figure 6. K.D.S. Initial Revenue Calculation - Correct Mathematical Modeling

In the business decision task, K.D.S. correctly built the geometric model and calculated total revenues (Figure 6). However, as shown in Figure 6, the reasoning disintegrated at the evaluative stage — K.D.S. claimed financial loss without derivation and failed to systematically incorporate the fixed operational costs that were central to the task.

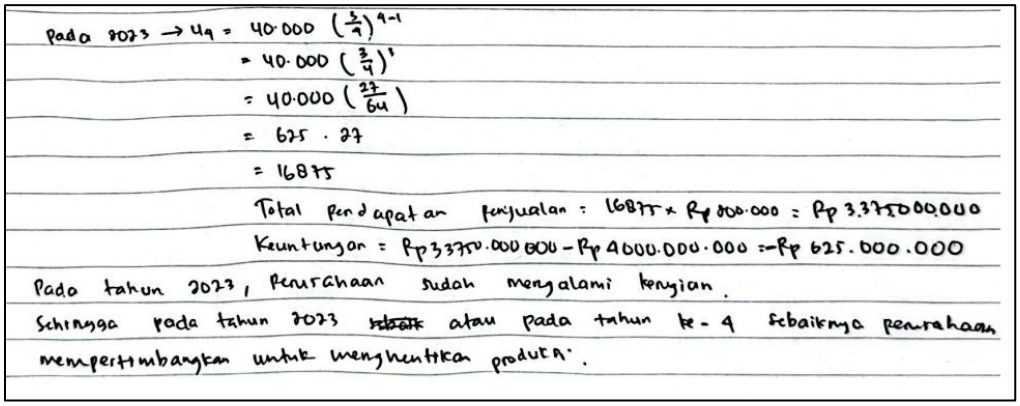

Figure 7. K.D.S. Contradictory Recommendations - Stop Production and Continue Calculations

The absence of self-correction is analytically important (See Figure 7). Procedural execution continued uninterrupted, despite the development of internal inconsistencies. This suggests that procedural completion acted as a cognitive stopping rule. The reasoning process was perceived as complete after conducting familiar procedures.

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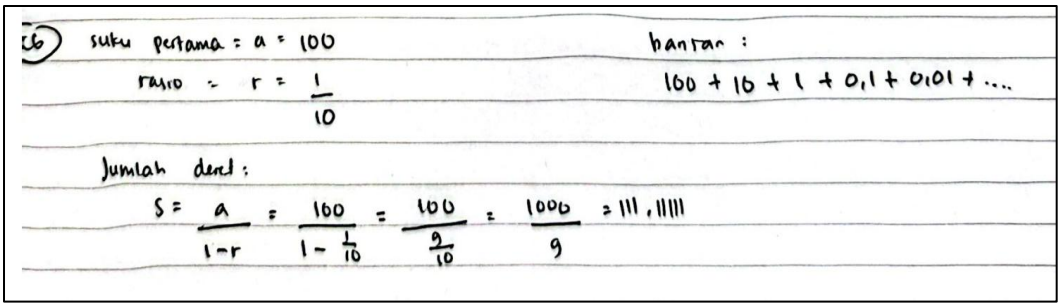

Figure 8. Student K.D.S. Response to Achilles Paradox, Example of Competence without Critical Interpretation

The same mechanism appears in the Achilles–tortoise task (Figure 8). K.D.S. correctly identified the infinite geometric series, applied the convergence formula, and obtained the correct numerical sum — yet left unanswered whether Achilles surpasses the tortoise. Since the task explicitly required this evaluative conclusion, the failure cannot be attributed to unclear instructions. Rather, once procedural execution yielded a satisfactory numerical result, reasoning was treated as complete — consistent with a procedural stopping rule in which procedural completion substitutes for genuine evaluative engagement. This reflects a deficit in critical thinking rather than modelling literacy: K.D.S. demonstrated sufficient knowledge to construct the correct model, but failed to connect the numerical output back to the question asked — a core dimension of critical thinking per the CCTST framework.

Procedural Validation and the Suppression of Critical Monitoring

The second mechanism concerns how procedural mastery redefines correctness criteria, discouraging critical monitoring. Procedurally dominant students justified answers through repetition rather than meaning, as illustrated by K.D.S. when asked how they checked their work.

> *"When I am unsure, I just keep trying again and again… if after trying several times the results are the same, then I think it is correct."*

This assertion points to a significant shift in epistemic standards. Justification, contextual fit, and logical consistency no longer function as criteria for correctness. However, correctness is determined by the stability of procedural output. The reasoning process is brought to a close when repeated execution yields the same result. This validation strategy helps explain the reason contradictions failed to trigger suspicion or revision. A related inhibitory effect is evident at the level of attentional control. In analyzing non-routine or contextual problems, K.D.S. stated that the process was guided by the search for procedural cues:

> *"I look for the ratio first… geometric series must have a ratio… usually the first term."*

The analysis reveals that procedural mastery shapes reasoning by prioritizing familiar processes while treating contextual constraints or missing parameters as impediments rather than meaningful indicators. Automated processes dominate cognitive resources, establish premature cessation conditions, and prevent access to evaluative and interpretive thinking. Students become

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procedurally stabilized, unable to engage with the demands of critical reasoning. By contrast, strong critical thinkers reported pausing, questioning procedural applicability, and purposefully reconsidering their results before reaching a conclusion.

"I had to stop just applying formulas and really think about what the patterns meant."

The shift to critical thinking is about restructuring the place of procedures in reasoning. Procedural mastery becomes counterproductive when reorganization fails to take place.

Discussion

This research demonstrates that the relationship between procedural mastery and critical thinking is operationally non-linear and under-characterized in current learning progression models. Although procedural mastery is widely assumed to support higher-order thinking, this assumption does not hold when students face tasks requiring interpretation, evaluation, and justification. While existing literature reports positive correlations between procedural fluency and mathematical achievement (Gilmore et al., 2017), these findings apply primarily to routine task contexts. The present results extend this picture by showing that procedural–conceptual performance can exhibit a weak negative correlation with critical thinking, while overall achievement is largely determined by critical reasoning. Procedural mastery is therefore not a reliable proxy for higher-order mathematical reasoning, helping explain why highly competent students encounter difficulties on non-routine tasks requiring judgment and interpretation.

The functional shift is accounted for by the expertise reversal effect. Traditionally, this theory explains why instructional aids become detrimental for advanced students (Kalyuga et al., 2012). The current results extend this theory by demonstrating that reversal can occur within students' reasoning processes even without explicit instructional changes. When procedural mastery becomes highly automatized, cognitive dominance directs attention, establishes validation standards, and governs halting rules — inhibiting executive monitoring and flexible thinking rather than freeing cognitive resources (Schoenfeld, 2016; Seufert, 2018).

The missing middle in score distributions provide further evidence that critical thinking cannot be achieved through gradual accumulation of procedural skills. The absence of intermediate performance levels supports threshold-based learning models, consistent with phenomenographic research reporting qualitatively different understanding patterns (Stott & Voutsina, 2024). Over-developed procedural automaticity stabilizes students in pre-threshold states, delaying the reasoning reorganization needed to cross the threshold — actively reinforcing modes of thought incompatible with critical engagement.

Qualitative data further clarify the mechanism. Procedurally dominant students accepted computational consistency as epistemic authority, terminating reasoning once a calculation was reached, consistent with research showing procedurally fluent students tend to avoid sense-making in mathematical justification and modeling (Levin, 2018). By contrast, students with strong critical thinking intentionally suspended routine practices and reframed results within mathematical meaning, indicating that the transition requires restructuring of procedural control.

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These findings suggest the existence of a procedural "sweet spot": insufficient procedural knowledge limits structural engagement, while excessive automaticity constrains analytical flexibility (Kalyuga et al., 2012). Three observable indicators help educators identify students approaching this boundary: ceiling-level procedural performance without self-initiated verification; restating procedural steps rather than offering contextual justification; and correct computation paired with missing evaluative responses — as observed in K.D.S. Targeted assessment designs separating procedural execution from interpretation items enable early identification of students at risk of procedural entrenchment.

The implications recommend a reconceptualisation of mathematics learning sequences in which procedural knowledge is understood as functionally variable. For procedurally expert students, two instructional principles are proposed. First, incorporating incorrect worked examples — procedurally correct but contextually flawed solutions — disrupts the procedural stopping rule by making the gap between computational accuracy and conceptual adequacy explicit. Second, embedding metacognitive prompts at key decision points, such as "What does this result mean in context?", externalises evaluative control that procedurally dominant students tend to bypass internally. Together, these strategies aim not to reduce procedural fluency but to subordinate it to structural interpretation and justificatory reasoning — repositioning procedures as tools for thinking rather than endpoints of reasoning.

Limitations and Future Directions

Several limitations constrain this research while pointing toward future directions. First, the single-cohort focus on geometric sequences limits empirical generalisation; future research should pursue theoretical replication across domains combining procedural affordances with interpretive demands, such as algebraic proof, calculus limits, and statistical modelling. Second, the cross-sectional design captures states rather than transitions; longitudinal research should trace movement from procedural dominance toward critical control. Third, instructional conditions were not manipulated; intervention research should test whether delaying automatization or emphasising procedural flexibility facilitates threshold crossing. Finally, pre-service teachers who have not completed the procedural-to-critical transition risk replicating procedurally dominant instructional patterns in their own classrooms, inadvertently signalling to secondary students that correct calculation constitutes complete mathematical thinking — creating a self-reinforcing cycle of procedural entrenchment across generations that future research should systematically examine.

Conclusion

In conclusion, procedural–conceptual mastery's effect on critical thinking was conditional, non-linear, and mechanism-dependent. Strong procedural performance did not reliably translate into critical reasoning, as evidenced by higher and more stable pre-critical scores compared to Critical Thinking performance, a negative procedural–critical association ($r = -0.34$), strong total–critical correlation ($r = 0.839$), and negligible total–procedural correlation ($r = 0.106$). The absence of intermediate cases — the "missing middle" — further confirmed that progression operated through qualitative thresholds rather than cumulative procedural increments.

Qualitative findings elaborated this structure by identifying a coherent cognitive mechanism. Students with high procedural fluency treated procedures as epistemic stopping rules, terminating

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reasoning upon reaching a computation rather than engaging in interpretation, evaluation, or justification. Progress occurred only when students reorganised reasoning control, subordinating procedures to structural interpretation and decision-making. These findings position procedural mastery as a dynamic resource whose function shifts according to task demands — supportive when guided by structural reasoning, but inhibitory when operating independently of it.

On pedagogical grounds, three recommendations follow for mathematics teacher education. First, assessment instruments should structurally separate procedural execution from interpretive and justificatory demands. Second, instructional sequences should deliberately introduce de-automatization activities after procedural mastery is established, requiring students to subordinate procedures to contextual judgment. Third, teacher educators should explicitly model the transition process — demonstrating not only correct procedure execution but also how to pause, question result sufficiency, and evaluate contextual meaning — to support pre-service teachers in completing the threshold transition before entering their own classrooms.

Declarations

AI Use Statement: During the preparation of this work, the authors used Claude AI (Anthropic) for language editing and literature search support. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Ethical Approval: This study was conducted in accordance with the Declaration of Helsinki and was approved by the Institutional Review Board of Universitas Terbuka, Indonesia (Protocol code 486/U31.LPPM/DI.04.01/2025).

Authors' Contributions: MN and TK performed the data collection; SN undertook the qualitative analysis; SS conceived the study, structured the conceptual framework, and coordinated the methodology integration; CS and HR reviewed the analysis and interpreted the data. All authors have read and agreed to the published version.

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